



# Failure mechanism dependence and reliability evaluation of non-repairable system



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## ABSTRACT

Reliability study of electronic system with the physics-of-failure method has been promoted due to the increase knowledge of electronic failure mechanisms. System failure initiates from independent failure mechanisms, have effect on or affect by other failure mechanisms and finally result in system failure. Failure mechanisms in a non-repairable system have many kinds of correlation. One failure mechanism developing to a certain degree will trigger, accelerate or inhibit another or many other failure mechanisms, some kind of failure mechanisms may have the same effect on the failure site, component or system. The destructive effect will be accumulated and result in early failure. This paper presents a reliability evaluation method considering correlativity among failure mechanisms, which includes trigger, acceleration, inhibition, and competition. Based on fundamental rule of physics of failure, decoupling methods of these correlations are discussed. With a case, reliability of electronic system is evaluated considering failure mechanism dependence.

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## 1. Introduction

Failure dependence has been extensively treated in reliability modeling for complicated system in aerospace, aviation, naval and nuclear power plants system. For example, a two-component parallel system, when one of the components fails, the stress places on the surviving component will change. Dependent failure will increase joint-failure probabilities, and then reduce system reliability [1]. Thus, for many complicated system, a modeling approach incorporating dependent failure resembles the true system reliability behavior in a more realistic manner.

Failure dependence exists in different levels of real-life system, these levels includes system level, component level and failure process or failure mechanism level. On system or component level, considerable research efforts have been devoted to modeling common cause failure (CCF), which is defined as a subset of dependent failures in which two or more component functional fault states exist at the same time, or within a short interval, as a result of a shared cause. For example, Ramirez-Marquez and Coit [2] proposed three different reliability optimization models for redundancy system subject to CCF and results show that the reconsideration of common cause failures will lead to different optimal design strategies.

Levitin [1] believes that CCFs of a system may cause by external cause as well as internal cause. Failures caused by common internal cause are called propagated failures. And for a component, the failure can be classified into local failures and propagated failures. Whether for a system or a component, propagated failure can be classified as global effect and selective effect [3]. Thus, propagated failures are common-cause-failures originated from a component of a system causing the failure of the entire system (global effect) or the failure of some of its sub-system (selective effect) [4].

Failure propagation may occur in sequence, which means input events occur in a prescribed order and results in the occurrence of the output event [5]. Global or selective effect of a propagated failure can be isolated in a system with function dependence behavior [6], which occurs when the failure of trigger component causes dependent component within the same system to become unusable or inaccessible. Failure isolation effect may take place in a system with function dependence. Xing and Levitin [3] defined this phenomenon as the trigger component fails before the propagated failure happens, in other words the failure propagation effect is prevented because another behavior happens. Thus there is the competition between propagation effect and isolated effect [3,6,7]. From this point, isolation effect is also associated with failure sequence.

Reliability of system subject to global effect and selective effect caused by imperfect fault coverage despite the presence of adequate redundancy and fault coverage has been studied for binary systems [3], multi-state systems [1] and ulti-trigger binary

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## Nomenclature

$M_i$	the $i$ th mechanism
$\Delta X$	accumulated damage in unit time
$t_i$	the failure time of $M_i$
$F(t)$	failure probability function of system
$F$	system failure
$F_i(t)$	failure probability function of $M_i$
$\zeta$	system lifetime

$f_i(t)$	failure density function of $M_i$
$X_{th}$	the threshold of system due to damage
$T_{tr}$	trigger time
$X_{Mitth}$	the threshold of damage caused by $M_i$
$F_{tr}(t)$	failure probability function of $T_{tr}$
$X_{M_i}(t)$	damage that $M_i$ brings to the system and varies with time $t$
$f_{tr}(t)$	failure density function of $T_{tr}$
$\Delta X_i$	damage in unit time due to $M_i$

systems [8]. Reliability analysis method of dynamic systems with sequence-dependent failures [5] and functional-dependent failures in phased-mission [9] are presented. Reliability and selective renewal policy of competing failures subject to failure isolation and propagation effects [3,7] are also studied.

Failure process or failure mechanism is the origin of component and system failure. Recently, Probabilistic Physics-of-Failure (PPoF) of system has been widely studied, which provides a greater understanding of failure mechanism, and also as an effective alternative to compensate for insufficient statistical failure data, offer another way to predict reliability [10]. Chookah [11] proposed a probabilistic Physics-of-Failure model for structures subject to pitting and corrosion-fatigue, which describes the probability density of structure degradation as the function of physical and critical environmental stresses and the corrosive parameters. Nagy [12] studied the joint uncertainty of the Arrhenius parameters, which enables uncertainty or reliability analysis for temperature dependence chemical kinetic systems.

Reliability modeling for systems with competing failure process has been investigated by several researchers, such as independent multiple catastrophic and degradation failure processes. Keedy and Feng [10,13] studied a stent with degradation and random shock failure mechanism, Probabilistic models for both failure processes are given and system reliability is acquired with the assumption that these two failure process is independent. Wang and Xing [8] studied the reliability of binary system, with the competition relationship in the time domain between failure isolation and propagation effect. Bocchetti [14] proposed a competing risk model to estimate the reliability of cylinder liners subject to two dominant failure modes, wear degradation and thermal cracking. Huang and Askin [15] presented reliability analysis of electronic devices with multiple competing failure modes involving catastrophic failures and degradation failures. With the probability that a product fails on a specific mode, the dominant failure mode on the product can be predicted.

In real-life world, failure mechanism dependent assumption is often violated in practice because environment and complicated system configuration all contribute to dependent failure mechanism. Slee [16] discussed propagating Printed Circuit Board (PCB) failure, which induced by different categories of failure mechanism including solder joint fatigue due to cyclic thermal mechanical stress, fatigue of plated-through holes contamination, electrochemical migration and the dependence of failure mechanism. Both of solder joint fatigue and plated-through holes fatigue can lead to resistive heating. And contaminants on a PCB can provide resistive and conductive path, which can cause propagating PCB failures. Peng [17] studied two dependent failure processes, which is called S-dependent [18]. These two dependent failure processes include soft failure caused by continuous degradation together with additional abrupt degradation damages due to a shock process, and hard failure caused by stress from the same shock process. They are correlated because the arrival of each shock load affects both failure processes and the shock process impacts the hard failure threshold level. His work has been extended by Jiang [18]

and Song [19] by considering reliability modeling of multiple components and changing, dependent failure threshold.

From the above discussion, previous studies on failure dependence are analyzed either from system level or from failure competing perspective. Methodologies for assessment of reliability of system incorporating these dependences have been widely investigated. However, little research has been conducted to explore failure mechanism dependent relationship and modeling and evaluating system reliability with failure mechanism and failure process data.

This paper identifies such issue of failure mechanism dependence or failure process dependence effect as competition, trigger, acceleration, inhibit and accumulation. Based on fundamental rule of physics of failure, we propose the decoupling process for these dependence effects. System reliability evaluation method for incorporation these dependence failure processes are investigated. Finally, in the case study, with the revised failure tree method, reliability of an electrical circuit with four components, thirteen failure mechanism and their competition, trigger, acceleration, accumulation effect are evaluated.

The remainder of the paper is organized as follows. Section 2 presents the classification of failure mechanism dependence correlations. Section 3 discusses five types of correlations, including competition, trigger, acceleration, inhibition and accumulation and their decoupling method. Section 4 presents a four-component electronic system with different failure mechanism correlations, and reliability evaluation of this system is discussed. Section 5 gives conclusions as well as directions for the future work.

## 2. Classification of failure mechanism correlation

From brewing, evolving and at last result in system failure, one failure mechanism will be influenced by other mechanisms due to the complication of system structure and loading condition. From engineering aspect, there are different types of failure mechanism correlations for non-repairable system, which are shown in Fig. 1.

Here, independent failure mechanism is defined as mechanisms only triggered by environmental condition, loading condition and inner parameters such as structure and material of failure parts. Independent failure mechanisms are not initiated, triggered or affected by any other failure mechanisms.

Some independent failure mechanisms have different development rates. System failure time will be determined by the failure

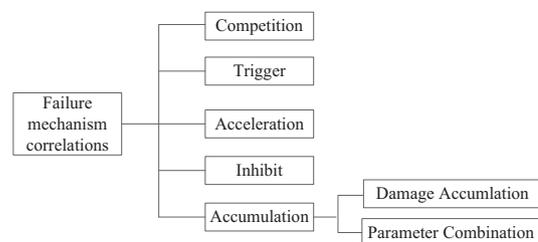


Fig. 1. Classification of failure mechanism correlations for non-repairable system.

time of the mechanism which develops to failure first. This process is competition, or these mechanisms have competition correlation.

One failure developing to a certain degree will lead to another or many other failure mechanisms, this type of correlation is called trigger, and the trigger event may be environmental or loading condition, may be other event happening suddenly.

One failure mechanism developing to a certain degree will accelerate (or inhibit) the development speed of other failure mechanisms, this correlation is called acceleration (or inhibition).

Some kind of failure mechanisms may have the same effect on the failure site, component or system. The destructive effect will be accumulated and result in early failure. These mechanisms have accumulation correlation. According to the different destructive ways, accumulation may be divided into damage accumulation and parameter combination. The former refers to mechanical damage. For example, in electronic interconnection part, both thermal fatigue and vibration fatigue will result in crack of solder joint. The accumulated damage determines lifetime of the solder joint.

Other than damage, failure mechanisms will also lead to the change of performance parameters. Some failure mechanisms act on the same part of the component, and result in the change of same parameters. Their correlation is parameter combination.

Failure mechanisms with these correlations intertwined with each other, make the reliability evaluation the electronic system very complicated. However, neglecting these correlations may lead to inaccurate evaluation results. To simplify this problem, decoupling methods of these correlations are necessary. The decoupling and modeling method is based on the following assumptions:

- 1) Both the system and its components are not repairable.
- 2) The failure process or mechanisms cannot be tested.
- 3) The system has two or more dependent binary-state failure mechanisms.
- 4) Failure mechanism of the system and their relationship can be identified and predicted.
- 5) Life distribution data of each failure mechanism can derive from PPoF analysis.

### 3. Reliability considering failure mechanism correlations

#### 3.1. Competition

Assume that there are  $n$  failure mechanisms, each mechanism is independent, and each will result in the failure of the complex system. Competition correlation is illustrated in Fig. 2. MACO (Mechanisms Competition) is the symbol of failure mechanisms competition correlation.

$F$  is system failure and  $M_i$  is the  $i$ th mechanism throughout the text,  $i = 1, 2, \dots, n$ .

Assume  $t_i$  to be the time of  $M_i$  from initiating to resulting in system failure,  $\zeta$  is system lifetime, then  $\zeta$  is the time of mechanism firstly resulting in system failure, as shown in the following

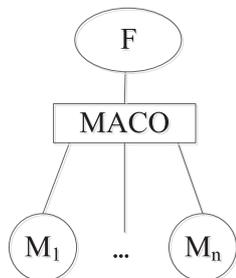


Fig. 2. Failure mechanisms competition.

formula:

$$\zeta = \min\{t_1, t_2, \dots, t_n\} \tag{1}$$

Assume  $X_{Mi}(t)$  indicates some kind of damage that  $M_i$  brings to the system and varies with time  $t$ , and  $X_{Mith}$  is the threshold of damage caused by  $M_i$ . When damage  $X_{Mi}$  increases to the threshold  $X_{Mith}$ , mechanism  $M_i$  will result in system failure. So system lifetime  $\zeta$  is

$$\zeta = \min\{\arg_t\{X_{Mi}(t) = X_{Mith}\}\} \tag{2}$$

In formula (1) and (2), failure time of a mechanism can be easily calculated by PoF equations. Many electronic failure mechanisms and their equations can be found in reference [20,21]. A general expression of failure time of a mechanism is

$$X(t) = f(D, M, S) \tag{3}$$

$X(t)$  represents failure time,  $D$  is design parameters,  $M$  is material parameters, and  $S$  is loads or environment stress.

For given failure mechanism, the dispersion of processing technic and environment or loading condition are taken into consideration, and failure distribution function of a mechanism  $f_i(t)$  can be obtained with the PPoF method [22–25].

$$f(t) = \text{PPoF}[X(t)] \tag{4}$$

Then failure probability of system  $F(t)$  is

$$\begin{aligned} F(t) &= P(\zeta \leq t) = 1 - P(\zeta > t) \\ &= 1 - P(t_1 > t, t_2 > t, \dots, t_n > t) \\ &= 1 - \prod_{i=1}^n [1 - P(t_i \leq t)] \\ &= 1 - \prod_{i=1}^n \left[ 1 - \int_0^t f_i(t) dt \right] \end{aligned} \tag{5}$$

#### 3.2. Trigger

Trigger correlation can be expressed in Fig. 3, where MACT (Mechanism Activate) is the symbol of failure mechanism trigger correlation.

When condition  $C_1$  is satisfied, the given failure mechanism  $M_a$  triggers  $M_i$  ( $i = 1, 2, \dots, n$ ) which are independent failure mechanisms. After trigger event,  $M_a$  is still active and not affected by  $M_i$ . Trigger time is  $T_{tr}$ , system lifetime  $\zeta$  can be expressed by

$$\zeta = \min\{t_a, T_{tr} + t_1, T_{tr} + t_2, \dots, T_{tr} + t_n\} \tag{6}$$

Failure probability of this system is

When  $t < T_{tr}$ ,

$$F(t) = F_a(t) \tag{7}$$

Assume  $f_i(t)$  is failure density function of  $M_i$ ,  $i = 1, 2, \dots, n$ .  $t_a$  is failure time of  $M_a$ , and  $f_a(t)$  is failure density function of  $M_a$ . After trigger event, i.e.  $t > T_{tr}$ , system failure probability is

$$\begin{aligned} F(t) &= 1 - R(t) \\ &= 1 - P(t_a > t, T_{tr} + t_1 > t, T_{tr} + t_2 > t, \dots, T_{tr} + t_n > t) \end{aligned}$$

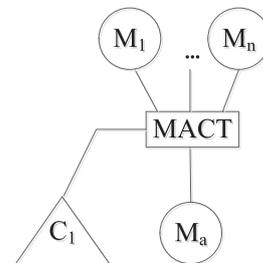


Fig. 3. Failure mechanism trigger.

$$\begin{aligned}
 &= 1 - [1 - P(t_a \leq t)] \prod_{i=1}^n [1 - P_i(T_{tr} + t_i \leq t)] \\
 &= 1 - [1 - F_a(t)] \prod_{i=1}^n [1 - F_i(t - T_{tr})] \\
 &= 1 - \left[ 1 - \int_0^t f_a(t) dt \right] \prod_{i=1}^n \left[ 1 - \int_0^{t-T_{tr}} f_i(t) dt \right] \tag{8}
 \end{aligned}$$

For failure mechanism  $M_i$  ( $i=1,2,\dots,n$ ), their failure density function  $f_i(t)$  ( $i=1,2,\dots,n$ ) can be obtained by the PPOF method.

Suppose that the trigger time  $T_{tr}$  is also distributed, with the failure probability function  $F_{tr}(t)$  and failure density function  $f_{tr}(t)$ , system failure probability at time  $t$  is

When  $t < T_{tr}$ ,

$$\begin{aligned}
 F(t) &= \Pr\{\text{trigger event will not happen at time } t\} \\
 &\quad \times \Pr\{\text{System fails}\} \\
 &= (1 - F_{tr}(t)) F_a(t) \\
 &= \left[ 1 - \int_0^t f_{tr}(t) dt \right] \int_0^t f_a(t) dt \tag{9}
 \end{aligned}$$

When  $t > T_{tr}$

$$\begin{aligned}
 F(t) &= \Pr\{\text{trigger event happen at time } T_{tr}\} \\
 &\quad \times \Pr\{\text{System fails}\} \\
 &= F_{tr}(T_{tr}) \left\{ 1 - [1 - F_a(t)] \prod_{i=1}^n [1 - F_i(t - T_{tr})] \right\} \\
 &= \int_0^{T_{tr}} f_{tr}(t) dt \left\{ 1 - \left[ 1 - \int_0^t f_a(t) dt \right] \prod_{i=1}^n \left[ 1 - \int_0^{t-T_{tr}} f_i(t) dt \right] \right\} \tag{10}
 \end{aligned}$$

### 3.3. Acceleration and inhibition

Acceleration and inhibition are illustrated in Fig. 4, where MACC (Mechanism Acceleration) is the symbol of mechanism acceleration, MINH (Mechanism Inhibition) is the symbol of mechanism inhibition.

A classic example of failure acceleration is an electronic module involves IC chips and other component. Heat dissipation of high power chips will accelerate the failure speed of adjacent components. Cases of inhibition correlation seem not easy to find. Mechanisms sensitive to high temperature will be accelerated by temperature elevation, while mechanisms sensitive to low temperature will be inhibited by temperature elevation. For example, rubber in high temperature is easy to become soft, which will inhibit embrittlement such as vitrification.

Acceleration or inhibition may have a trigger event or not, it depends on the mechanisms.

There are  $n$  failure mechanisms  $M_1, M_2, \dots, M_n$  in system. Independently, all of them will result in the failure of the whole system. Under condition 1, their respective reliability are  $R_1, \dots, R_n$ . While under condition 2, these mechanisms will be accelerated or inhibited, their development speed will change, in order to distinguish, use  $M' 1, M' 2, \dots, M' n$  to represent these failure mechanisms. Assume that under condition 2, their failure time are  $t' 1, t' 2, \dots, t' n$ .

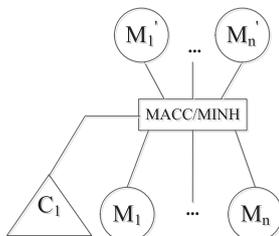


Fig. 4. Failure mechanism acceleration or inhibition.

When event  $C_1$  happens at time  $T_{tr}$ , condition 1 will change to condition 2,  $M_1, M_2, \dots, M_n$  will be accelerated or inhibited, but they are still independent. Then, system lifetime is

$$\zeta = \min\{T_{tr} + t_{r1}, T_{tr} + t_{r2}, \dots, T_{tr} + t_{rn}\} \tag{11}$$

where  $t_{r1}, t_{r2}, \dots, t_{rn}$  are residual lifetime of  $M_1, M_2, \dots, M_n$  after event  $C_1$  happens. Suppose the development process of these mechanisms are linearly, that means the development speed is constant, then

$$T_{ri} = \left( 1 - \frac{T_{tr}}{t_i} \right) \times t'_i \quad i = 1, \dots, n \tag{12}$$

Then system failure probability is

$$\begin{aligned}
 F(t) &= P(\zeta \leq t) = 1 - P(\zeta > t) \\
 &= 1 - P(T_{tr} + t_{r1} > t, \dots, T_{tr} + t_{rn} > t) \\
 &= 1 - \prod_{i=1}^n [1 - F_{ri}(t - T_{tr})] \\
 &= 1 - \prod_{i=1}^n \left[ 1 - \int_0^{t-T_{tr}} f_{ri}(t) dt \right] \tag{13}
 \end{aligned}$$

Suppose  $T_{tr}$  is distributed, with the failure probability function of  $F_{tr}(t)$ , system failure probability at time  $t$  is

When  $t < T_{tr}$

$$F(t) = [1 - F_{tr}(t)] \left\{ 1 - \prod_{i=1}^n [1 - F(t_i)] \right\} \tag{14}$$

When  $t > T_{tr}$

$$F(t) = F_{tr}(T_{tr}) \left\{ 1 - \prod_{i=1}^n [1 - F_i(t - T_{tr})] \right\} \tag{15}$$

### 3.4. Accumulation

Fig. 5 illustrates failure mechanism accumulation correlation, where  $M_1, \dots, M_n$  are failure mechanisms and  $F$  is their common consequence. According to the destructive type, accumulation can be divided into damage accumulation (MADA, Mechanism damage accumulation) and parameter combination (MAPA, Mechanism Parameter combination).

$M_i$  ( $i=1,2,\dots,n$ ) act on the same part of system and result in the same kind of damage. Damage accumulating to a certain extent will result in failure of the whole system. If  $M_1, \dots, M_n$  have damage accumulation correlation, the threshold of this system due to this kind of damage is  $X_{th}$ , then

$$\Delta X_i = \frac{X_{th}}{t_i} \tag{16}$$

Where  $\Delta X_i$  is the damage in unit time due to  $M_i$ ,  $t_i$  is the failure time due to  $M_i$  when it works alone. Then lifetime of system is

$$\begin{aligned}
 \zeta &= \frac{X_{th}}{\Delta X} = \frac{X_{th}}{\lambda_1 \Delta X_1 + \dots + \lambda_i \Delta X_i} \\
 &= \frac{X_{th}}{\lambda_1 \frac{X_{th}}{t_1} + \dots + \lambda_i \frac{X_{th}}{t_i}} \\
 &= \frac{1}{\frac{\lambda_1}{t_1} + \dots + \frac{\lambda_n}{t_n}} = \frac{1}{\sum_{i=1}^n \frac{\lambda_i}{t_i}} \tag{17}
 \end{aligned}$$

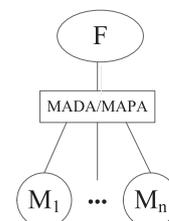


Fig. 5. Accumulation correlation.

where  $\Delta X$  is the accumulated damage in unit time. And  $\lambda_i$  is a scaling factor of  $M_i$ ,  $i=1,2,\dots,n$ .

System failure probability  $F(t)$  is

$$F(t) = P(\zeta \leq t) = P\left(\frac{1}{\sum_{i=1}^n \frac{\lambda_i}{t_i} \leq t}\right) \tag{18}$$

The scaling factors should be determined according to the different failure mechanisms. In practice, the accumulation of different mechanisms often supposed to be linear refers to Miner's rule.

**4. A case study**

An electrical system is shown in Fig. 6, which is composed of four components, two integrated circuits IC<sub>1</sub> and IC<sub>2</sub>, one multi-layer ceramic capacitor C<sub>1</sub> and one transistor V<sub>1</sub>. They are all assembled on one Printed Circuit Board.

Failure mechanisms of this system are shown in Table 1. Working environment condition includes temperature cycle and random vibration. Solder joint of these components will have thermal fatigue and vibration fatigue mechanism. In Table 1, VF is vibration fatigue, TF is thermal fatigue. TDDDB is time-dependent dielectric breakdown, NBTI is negative bias temperature instability, EM is electrical migration, SC is crack due to shock, EB is electrical break and DE is PCB deformation.

A kind of mechanism can lead several effects and different mechanisms can lead to the same effect. Symbols in the third column represent respective mechanisms in the second column. For example, Af<sub>1</sub> represents VF of IC<sub>1</sub> and Cf<sub>1</sub> represents SC of C<sub>1</sub>. Symbols in the fifth column represent respective failure effects in the sixth column. For example, Ma<sub>1</sub> represents IC<sub>1</sub> solder open and Mc<sub>3</sub> represents IC<sub>2</sub> EM acceleration.

Life distributions of the mechanism listed in Table.1 are given in Table 2.

In this case, 1000 random numbers are generated from each corresponding life distribution of mechanism. Reliability functions of components or system considering failure mechanism dependence are achieved based on these data.

**4.1. Failure mechanism correlation of IC<sub>1</sub>**

Failure mechanisms of IC<sub>1</sub> have the following correlation, damage accumulation of VF and TF, parameter combination of TDDDB and NBTI and competition of their results, which is illustrated in Fig. 7.

**4.1.1. Damage accumulation of VF and TF**

IC<sub>1</sub> solder VF(Af<sub>1</sub>) and TF(Af<sub>2</sub>) mechanism will both result in solder crack. Their damage result will accumulate and finally result in IC<sub>1</sub> open failure due to solder crack, which is illustrated in Fig. 8.

Suppose the damage threshold is Xth, t<sub>Af<sub>1</sub></sub> is the failure time due to Af<sub>1</sub>, and t<sub>Af<sub>2</sub></sub> is the failure time to Af<sub>2</sub>. Then

$$\Delta X_{Af_1} = \frac{Xth}{t_{Af_1}}, \Delta X_{Af_2} = \frac{Xth}{t_{Af_2}} \tag{19}$$

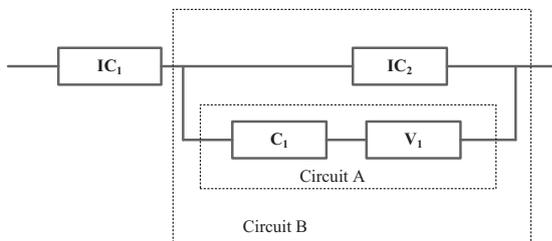


Fig. 6. Electrical system.

**Table 1**  
Failure mechanisms of electrical system.

Component	Mechanism	Mechanism symbol	Failure effect	Effect symbol
IC <sub>1</sub>	VF	Af <sub>1</sub>	IC <sub>1</sub> solder open	Ma <sub>1</sub>
	TF	Af <sub>2</sub>	IC <sub>1</sub> solder open	Ma <sub>1</sub>
	TDDDB	Af <sub>3</sub>	IC <sub>1</sub> Parameter drift	Ma <sub>2</sub>
	NBTI	Af <sub>4</sub>	IC <sub>1</sub> parameter drift	Ma <sub>2</sub>
IC <sub>2</sub>	VF	Bf <sub>1</sub>	IC <sub>2</sub> solder open	Mb <sub>1</sub>
	TF	Bf <sub>2</sub>	IC <sub>2</sub> solder open	Mb <sub>1</sub>
	EM	Bf <sub>3</sub>	IC <sub>2</sub> chip open	Mb <sub>2</sub>
C <sub>1</sub>	SC	Cf <sub>1</sub>	C <sub>1</sub> open	Mc <sub>1</sub>
			Circuit A open	Mc <sub>2</sub>
			IC <sub>2</sub> EM acceleration	Mc <sub>3</sub>
V <sub>1</sub>	VF	Df <sub>1</sub>	V <sub>1</sub> solder open	Md <sub>1</sub>
			Circuit A open	Mc <sub>2</sub>
			IC <sub>2</sub> EM acceleration	Mc <sub>3</sub>
	TF	Df <sub>2</sub>	V <sub>1</sub> solder open	Md <sub>1</sub>
			Circuit A open	Mc <sub>2</sub>
			I <sub>2</sub> EM acceleration	Mc <sub>3</sub>
PCB	EB	Df <sub>3</sub>	V <sub>1</sub> open	Md <sub>2</sub>
	DE	Ef <sub>1</sub>	C <sub>1</sub> crack	Me <sub>1</sub>

**Table 2**  
Life distribution of mechanisms in case study.

Mechanism symbol	Distribution type	Characteristic parameter		
		$\beta(\theta)$	$\eta(\sigma)$	$\lambda$
Af <sub>1</sub>	Weibull	3.28	7620	/
Af <sub>2</sub>	Weibull	2.33	9211	/
Af <sub>3</sub>	Lognormal	9.69	0.31	/
Af <sub>4</sub>	Lognormal	8.92	0.27	/
Bf <sub>1</sub>	Weibull	2.94	6509	/
Bf <sub>2</sub>	Weibull	2.33	8230	/
Bf <sub>3</sub>	Weibull	3.17	3490	/
Df <sub>1</sub>	Weibull	1.85	7090	/
Df <sub>2</sub>	Weibull	2.33	9012	/
Df <sub>3</sub>	Weibull	2.85	5490	/
Ef <sub>1</sub>	Exponential	/	/	3970

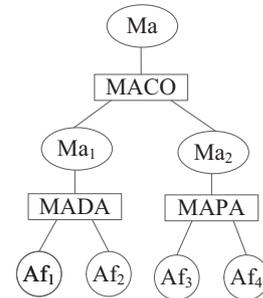


Fig. 7. Failure mechanism correlation for IC<sub>1</sub>.

where  $\Delta X_{Af_1}$  is the fatigue crack length in unit time due to Af<sub>1</sub>,  $\Delta X_{Af_2}$  is the fatigue crack length in unit time due to Af<sub>2</sub>. Then

$$t_{Ma_1} = \frac{Xth}{\Delta X} = \frac{Xth}{\lambda_1 \Delta X_{Af_1} + \lambda_2 \Delta X_{Af_2}} = \frac{t_{Af_1} t_{Af_2}}{\lambda_2 t_{Af_1} + \lambda_1 t_{Af_2}} \tag{20}$$

Where  $\Delta X$  is the accumulated damage in one thermal cycle. And  $\lambda_1, \lambda_2$  are scaling factors of thermal cycle and vibration cycle.  $t_{Ma_1}$  is the failure time due to accumulated mechanism Af<sub>1</sub> and

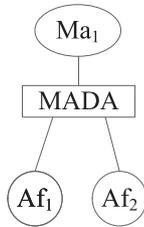


Fig. 8. VF and TF damage accumulation of IC<sub>1</sub>.

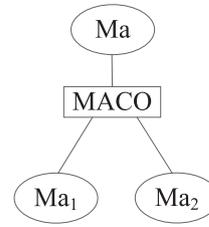


Fig. 10. Competition between Ma<sub>1</sub> (damage accumulation of Af<sub>1</sub> and Af<sub>2</sub>) and Ma<sub>2</sub> (parameter combination of Af<sub>3</sub> and Af<sub>4</sub>).

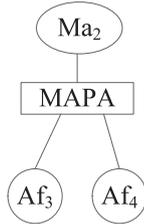


Fig. 9. TDDB and NBTI parameter combination of IC<sub>1</sub>.

mechanism Af<sub>2</sub>. Here we suppose the thermal fatigue crack and vibration fatigue crack accumulated linearly.

Failure density function of Af<sub>1</sub> is  $f_{Af_1}(t)$ , and Af<sub>2</sub> is  $f_{Af_2}(t)$ , which can be obtained by PPOF process of their failure physics equations. In this case, PPOF process is neglected, and failure distribution functions are given directly in Table 2. Failure probability function of Ma<sub>1</sub> can be achieved by Eqs. (18)–(20).

For other damage accumulated mechanism correlations in this case, same method can be used to acquire their failure probability density function.

#### 4.1.2. Parameter combination of TDDB and NBTI

IC<sub>1</sub> TDDB(Af<sub>3</sub>) and NBTI(Af<sub>4</sub>) mechanism will both result in the increase of response delay time. Suppose the accumulated response delay time exceeding the threshold Pth will result in IC<sub>1</sub> permanent failure due to temporal chaos, which is illustrated in Fig. 9

Lifetime for parameter combination of mechanism Af<sub>3</sub> and Af<sub>4</sub> is

$$t_{Ma_2} = \frac{t_{Af_3} \times t_{Af_4}}{t_{Af_3} + t_{Af_4}} \quad (21)$$

Where  $t_{Af_3}$  and  $t_{Af_4}$  are failure time due to mechanism Af<sub>3</sub> and Af<sub>4</sub>. Given the failure density function of IC<sub>1</sub> mechanism Af<sub>3</sub> and Af<sub>4</sub>, failure probability function of Ma<sub>2</sub> can be obtained.

#### 4.1.3. Failure competition

Competition correlation between Ma<sub>1</sub> and Ma<sub>2</sub> is illustrated in Fig. 10.

IC<sub>1</sub> solder open and parameter drift mechanism will compete, and the results depend on whose failure time is shorter. Based on the above description, failure time of IC<sub>1</sub> can be calculated as

$$t_{Ma} = \min\{t_{Ma_1}, t_{Ma_2}\} \\ = \min\left\{\frac{t_{Af_1} \times t_{Af_2}}{t_{Af_1} + t_{Af_2}}, \frac{t_{Af_3} \times t_{Af_4}}{t_{Af_3} + t_{Af_4}}\right\} \quad (22)$$

Under the assumption that these failure mechanisms are independent, failure time of IC<sub>1</sub> can be obtained as

$$t'_{Ma} = \min\{t_{Af_1}, t_{Af_2}, t_{Af_3}, t_{Af_4}\} \quad (23)$$

It is easy to prove that

$$t_{Ma_1} < t_{Af_1}, \text{ and } t_{Ma_1} < t_{Af_2}$$

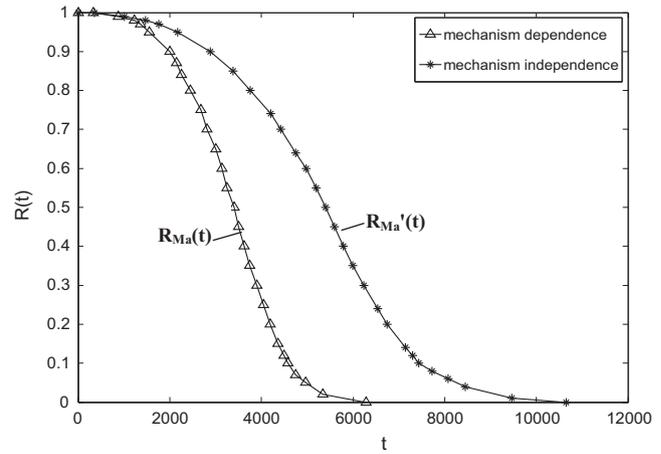


Fig. 11. Reliability function of Ma (Component IC<sub>1</sub> failure).

$$t_{Ma_2} < t_{Af_3}, \text{ and } t_{Ma_2} < t_{Af_4}$$

$$\text{So } t_{Ma} < t'_{Ma}$$

Failure time data of IC<sub>1</sub> is achieved based on the failure time data of mechanism Af<sub>1</sub>, Af<sub>2</sub>, Af<sub>3</sub> and Af<sub>4</sub>. For example, when  $t_{Af_1}=6328$ ,  $t_{Af_2}=4763$ ,  $t_{Af_3}=7401$  and  $t_{Af_4}=5394$ , based on the equations (22) and (23),  $t_{Ma}=2717.5$  and  $t'_{Ma}=4763$ . Cumulative distribution function (CDF) of failure time (i.e, failure probability function) of IC<sub>1</sub> can be obtained. Then reliability function is easy to draw.

Reliability of IC<sub>1</sub> considering mechanism dependence is shown in Fig. 11, compared with the condition that mechanisms are independent. Ma is IC<sub>1</sub> failure,  $R_{Ma}(t)$  is defined as the reliability of IC<sub>1</sub> considering mechanism dependence, while  $R'_{Ma}(t)$  is reliability of IC<sub>1</sub> with mechanism independence assumption.

From Fig. 11, failure time of IC<sub>1</sub> considering mechanism accumulation dependence is obviously shorter than they are independence. Seen from Fig. 10, at time  $t$ , reliability considering dependence  $R_{Ma}(t)$  is less than  $R'_{Ma}(t)$ , reliability with mechanism independence assumption.

#### 4.2. Failure mechanism of C<sub>1</sub>

PCB deformation will trigger the crack of multi-layer ceramic capacitor C<sub>1</sub>, and will result in Mc<sub>1</sub>, C<sub>1</sub> open failure, which is illustrated in Fig. 12.

Assume vibration shock happens at time  $T_{tr}=2400$  h, Ef<sub>1</sub> is exponentially distributed as given in Table 2, from Eqs. (6) and (8), the CDF of Cf<sub>1</sub> failure time will be obtained.

Based on decoupling method above, failure time of C<sub>1</sub> can be obtained as

$$t_{Mc_1} = \min\{t_{Ef_1}, T_{tr} + t_{rc_1}\} \quad (24)$$

In this case, shock will directly result in the crack of C<sub>1</sub>. So rest failure time of C<sub>1</sub> after shock  $t_{rc_1} = 0$ .

Under the assumption that these failure mechanisms are independent, failure time of  $C_1$  can be obtained as

$$t'_{Mc_1} = \min\{t_{Ef_1}, T_{tr}\} \tag{25}$$

So, in this case  $t_{Mc_1} = t'_{Mc_1}$ .

Reliability of  $C_1$  considering mechanism dependence is shown in Fig. 13, compared with the condition that mechanisms are independent.

Because trigger event shock will directly trigger the crack of  $C_1$ , and there is no difference between failure time of  $C_1$  considering mechanism dependence or not. The turning point in Fig. 13 is because of the shock at the trigger time of 2400 h, which result in the failure of  $C_1$  directly.

### 4.3. Failure mechanism correlation of $V_1$

Failure mechanism of  $V_1$  and its mechanism correlation is illustrated in Fig. 14, in which the damage due to mechanism  $Df_1$  and  $Df_2$  will be accumulated and compete with mechanism  $Df_3$ .  $Md$  is the result of competition, if  $Md_1$  happens earlier than  $Md_2$ , then  $Md$  is  $Md_1$ , otherwise  $Md$  is  $Md_2$ . Failure result of  $Md_1$  and  $Md_2$  are all  $V_1$  open, which is expressed by  $Md$ .

According to the description in Section 4.1, failure time of  $V_1$  is

$$t_{Md} = \min\{t_{Md_1}, t_{Md_2}\} \\ = \min\left\{\frac{t_{Df_1} \times t_{Df_2}}{t_{Df_1} + t_{Df_2}}, t_{Df_3}\right\} \tag{26}$$

While

$$t'_{Md} = \min\{t_{Df_1}, t_{Df_2}, t_{Df_3}\} \tag{27}$$

So  $t_{Md} \leq t'_{Md}$ .

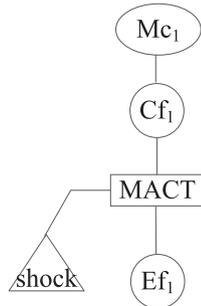


Fig. 12. Failure tree of  $C_1$ .

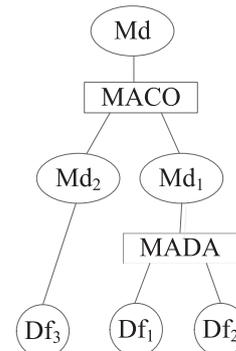


Fig. 14. Failure tree of  $V_1$ .

Reliability of  $V_1$  considering mechanism dependence is shown in Fig. 15, compared with the condition that mechanisms are independent.

From Fig. 15, because of parameter combination between  $Df_1$  and  $Df_2$ , failure time of  $V_1$  considering mechanism dependence is obviously shorter than that without this consideration. And at time  $t$ , reliability considering dependence  $R_{Md}(t)$  is less than  $R_{Md'}(t)$  with independence assumption.

### 4.4. Failure competition of $C_1$ and $V_1$

$C_1$  open and  $V_1$  open have competition correlation. Whether  $C_1$  open occurs earlier or  $V_1$  open occurs earlier will lead to circuit A open, which is expressed by  $Mc$ ,  $Mc_2$  is also the trigger event of  $IC_2$  EM mechanism acceleration. Failure competition of  $C_1$  and  $V_1$  is illustrated in Fig. 16.

So failure time of circuit A open is obtained as

$$t_{Mc_2} = \min\{t_{Mc_1}, t_{Md}\} \tag{28}$$

While

$$t'_{Mc_2} = \min\{t'_{Mc_1}, t'_{Md}\} \geq \min\{t_{Mc_1}, t_{Md}\} \tag{29}$$

So  $t_{Mc_2} \leq t'_{Mc_2}$ .

Reliability of circuit A considering mechanism dependence is shown in Fig. 17, compared with the condition that mechanisms are independent.

Circuit A consists of  $C_1$  and  $V_1$ . Failure of  $C_1$  or  $V_1$  will lead to failure of circuit A.  $C_1$  open and  $V_1$  open will compete with each other. Because of mechanism dependence, failure time of  $V_1$  is

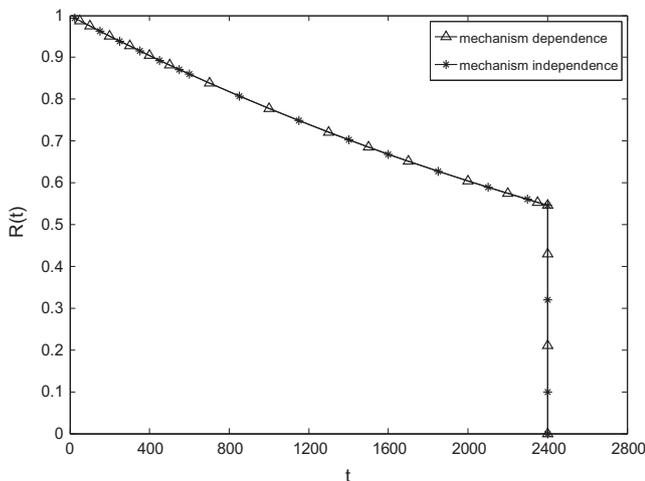


Fig. 13. Reliability function of  $Mc_1$  (Component  $C_1$  failure).

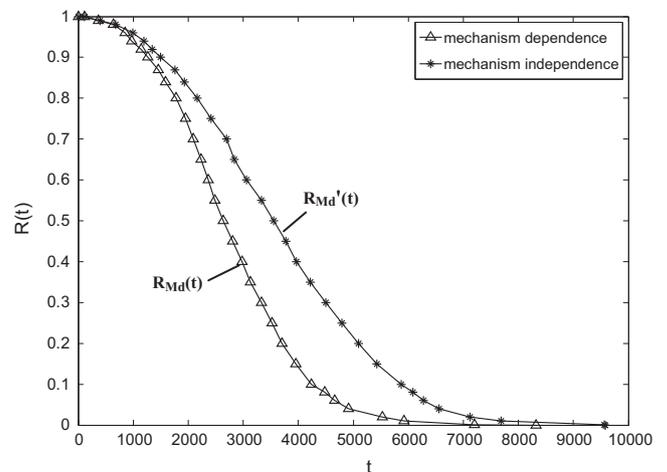


Fig. 15. Reliability function of  $Md$  (Component  $V_1$  failure).

shorter and more likely less than failure time of  $C_1$ . Then  $V_1$  failure will contribute more to failure of circuit A. As seen from Fig. 17, at time  $t$  ( $0 < t < 2400$ ), reliability with mechanism dependence consideration is less than reliability without dependence consideration. When  $t=2400$  h,  $C_1$  fails directly because of shock, which will lead to failure of circuit A. So at  $t=2400$  h, the reliability function will suddenly drop to zero, and turning point a's corresponding reliability is smaller than that of turning point b.

4.5. Failure acceleration of  $IC_2$  EM mechanism

$C_1$  open or  $V_1$  open will result in circuit A open, and will further increase current in  $IC_2$  and then  $IC_2$  EM mechanism will be accelerated and finally shorten the failure time of  $IC_2$  open, which is illustrated in Fig. 18.

Given  $I_1$  and  $I_2$  are the current density of metallic interconnects before acceleration and after acceleration, where EM mechanism will happen and the failure time of  $IC_2$  open is  $t_{Mc_2}$

$$t_{Mb_2} = \begin{cases} t_{Bf_3} & t_{Bf_3} \leq t_{Mc_2} \\ t_{Mc_2} + (1 - \frac{t_{Mc_2}}{t_{Bf_3}})t_{Bf_3}' & t_{Bf_3} > t_{Mc_2} \end{cases} \quad (30)$$

$t_{Bf_3}$  is the failure time of  $IC_2$  due to mechanism  $Bf_3$  when current is  $I_1$ .  $t_{Bf_3}'$  is the failure time of  $IC_2$  due to mechanism  $Bf_3$  when current is  $I_2$ .  $t_{Mb_2}$  is the failure time of  $IC_2$  due to EM mechanism, before  $t_{Mc_2}$ , the current is  $I_1$  and after  $t_{Mc_2}$ , the current is  $I_2$ . And the failure density function of EM failure acceleration can be given by Eqs. (12) and (15).

Under the assumption of failure mechanisms independence, failure time of  $IC_2$  due to EM mechanism is

$$t'_{Mb_2} = t_{Bf_3}.$$

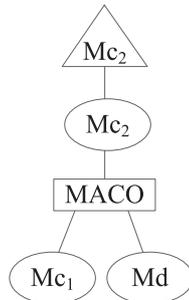


Fig. 16. Competition of  $C_1$  open and  $V_1$  open.

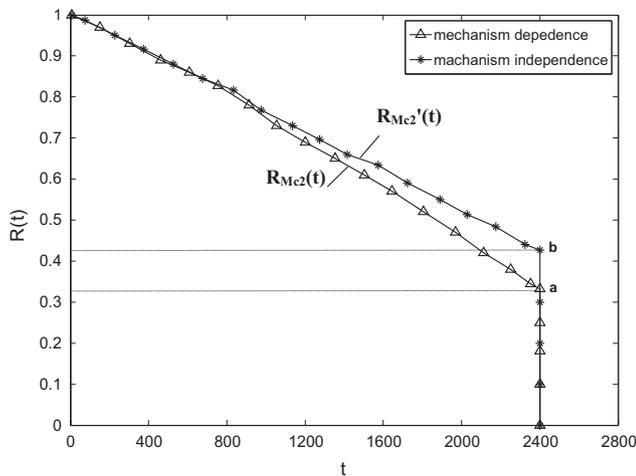


Fig. 17. Reliability function of  $Mc_2$  (Circuit A failure).

Because circuit A open will accelerate  $IC_2$  EM mechanism, then the failure time

$$t_{Bf_3}' < t_{Bf_3}.$$

When  $t_{Bf_3} \leq t_{Mc_2}, t_{Mb_2} = t_{Bf_3};$

When  $t_{Bf_3} > t_{Mc_2},$

$$t_{Mb_2} = t_{Mc_2} + (1 - \frac{t_{Mc_2}}{t_{Bf_3}})t_{Bf_3}' < t_{Mc_2} + (1 - \frac{t_{Mc_2}}{t_{Bf_3}})t_{Bf_3} = t_{Bf_3}$$

So  $t_{Mb_2} \leq t_{Bf_3} = t'_{Mb_2}.$

Reliability of  $IC_2$  considering mechanism dependence is shown in Fig. 19, compared with the condition that mechanisms are independent.

Circuit A open will accelerate  $IC_2$  EM mechanism. As seen from Fig. 19, reliability of  $IC_2$  EM mechanism considering mechanism dependence,  $R_{Mb_2}(t)$  is less than  $R_{Mb_2'}(t)$ , which is reliability with independence assumption.

4.6. Failure mechanism correlation of  $IC_2$

Failure mechanisms of  $IC_2$  have the following correlation, damage accumulation of  $VF(Bf_1)$  and  $TF(Bf_2)$ , competition of solder open( $Mb_1$ ) and chip open( $Mb_2$ ) due to EM mechanism( $Bf_3$ ), which is illustrated in Fig. 20.  $Mb$  is  $IC_2$  failure.

The competition result of  $IC_2$  solder open and  $IC_2$  chip open is  $IC_2$  open.

According the description above, failure time of  $IC_2$  open is obtained as

$$t_{Mb} = \min\{t_{Mb_1}, t_{Mb_2}\}$$

$$= \min\left\{\frac{t_{Bf_1} \times t_{Bf_2}}{t_{Bf_1} + t_{Bf_2}}, t_{Bf_3}\right\} \quad (31)$$

while

$$t'_{Mb} = \min\{t_{Bf_1}, t_{Bf_2}, t_{Bf_3}\} \quad (32)$$

So  $t_{Mb} \leq t'_{Mb}.$

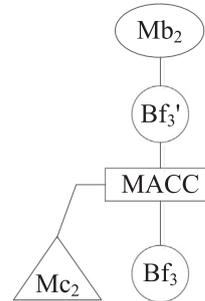


Fig. 18. Failure acceleration of  $IC_2$  EM.

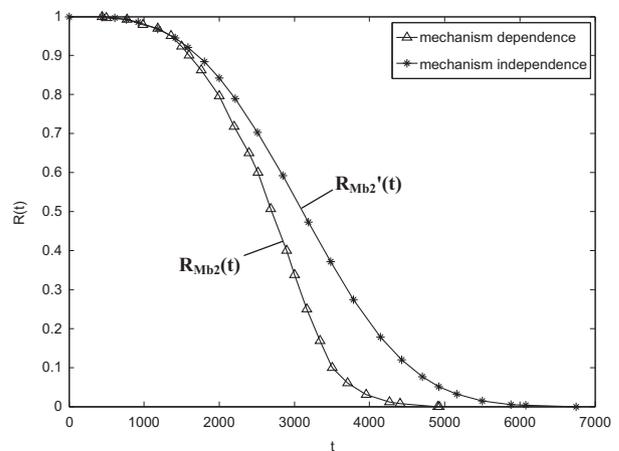


Fig. 19. Reliability function of  $Mb_2$  ( $IC_2$  EM mechanism).

Reliability of IC<sub>2</sub> considering mechanism dependence is shown in Fig. 21, compared with the condition that mechanisms are independent.

With the influence of Mb<sub>2</sub> and parameter combination between Bf<sub>1</sub> and Bf<sub>2</sub>, failure of IC<sub>2</sub> is accelerated. Range of failure time with mechanism dependence is more concentrated than that with independence assumption. Therefore, reliability of IC<sub>2</sub> considering mechanism dependence, R<sub>Mb</sub>(t) function curve is more steeper than R<sub>Mb'</sub>(t), which is reliability of IC<sub>2</sub> with independence assumption.

4.7. Failure correlation of circuit A and IC<sub>2</sub>

Circuit A and IC<sub>2</sub> are branches in the parallel circuit. When both circuit A and IC<sub>2</sub> fail, circuit B will fail. In this paper, AND is used to illustrate failure correlation of parallel circuit, which is illustrated in Fig. 22.

$$t_{Mc} = \max\{t_{Mb}, t_{Mc_2}\} \tag{33}$$

While

$$t'_{Mc} = \max\{t'_{Mb}, t'_{Mc_2}\} \geq \max\{t_{Mb}, t_{Mc_2}\} \tag{34}$$

so  $t_{Mc} \leq t'_{Mc}$

Reliability of circuit B considering mechanism dependence is shown in Fig. 23, compared with the condition that mechanisms are independent.

Occurrence of the turning points is because of trigger event. Failure time decreases considering mechanism dependence in this case, so probability of failure before the trigger event happen increases. Therefore, turning point a's corresponding reliability is smaller than that of turning point b, as shown in Fig. 23.

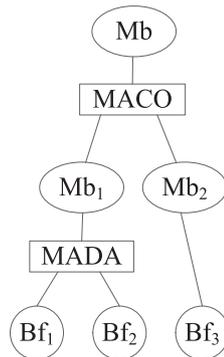


Fig. 20. IC<sub>2</sub> failure mechanism tree.

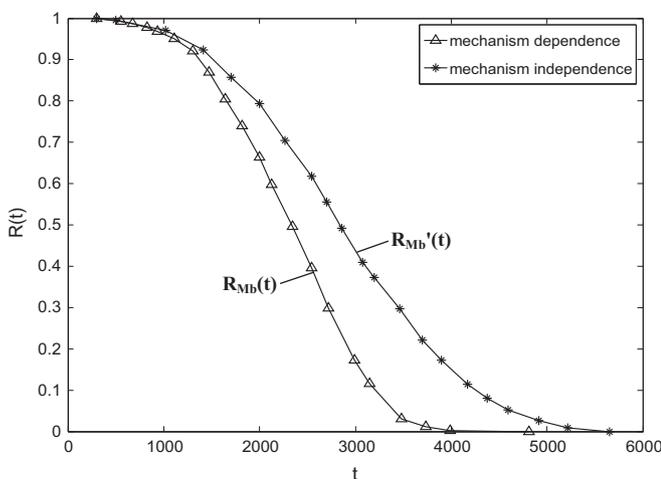


Fig. 21. Reliability function of Mb (Component IC<sub>2</sub> failure).

4.8. Failure mechanism correlation of IC<sub>1</sub> and circuit B

Finally IC<sub>1</sub> failure and circuit B failure will compete and result in system failure which is illustrated in Fig. 24.

Known from Fig. 24, system failure time  $\zeta$  is,

$$\zeta = \min\{t_{Ma}, t_{Mc}\} \tag{35}$$

Known from the above

$$t_{Ma} < t'_{Ma}, \text{ and } t_{Mc} \leq t'_{Mc}$$

While

$$\zeta' = \min\{t'_{Ma}, t'_{Mc}\} \geq \min\{t_{Ma}, t_{Mc}\} = \zeta \tag{36}$$

So  $\zeta \leq \zeta'$ .

The comparison of the reliability function of system considering mechanism dependence or not is shown in Fig. 25.

Because of mechanism dependence, failure time of components decrease. Then failure time of system will decrease consequently, which is shown in Fig. 25.

4.9. Reliability evaluation of system

Failure mechanism tree of Fig. 6 according to the analysis above is illustrated in Fig. 26.

Failure time of system considering failure mechanism dependence and under the independent mechanism assumption is achieved with simulation.

The comparison of the reliability function of reliability function of IC<sub>1</sub>, circuit B and the system is shown in Fig. 25.

IC<sub>1</sub> failure and circuit B failure will compete and result in system failure, so

$$R_M(t) = R_{Ma}(t)R_{Mc}(t)$$

As shown in Fig. 27(b), with mechanism independence assumption, R<sub>Ma</sub>(t) decreases slowly and smoothly with time. While R<sub>Mc</sub>(t) decreases more rapidly and has turning point because of shock event. Therefore, tendency of R<sub>M</sub>(t) is more affected by R<sub>Mc</sub>(t).

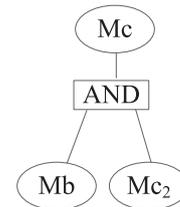


Fig. 22. Failure correlation of circuit A and circuit B.

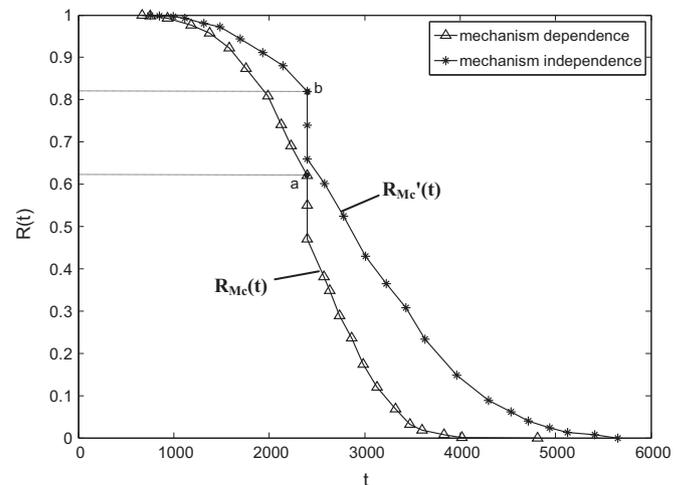


Fig. 23. Reliability function of Mc (circuit B).

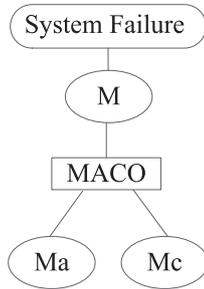


Fig. 24. Failure competition of IC<sub>1</sub> and IC<sub>2</sub>.

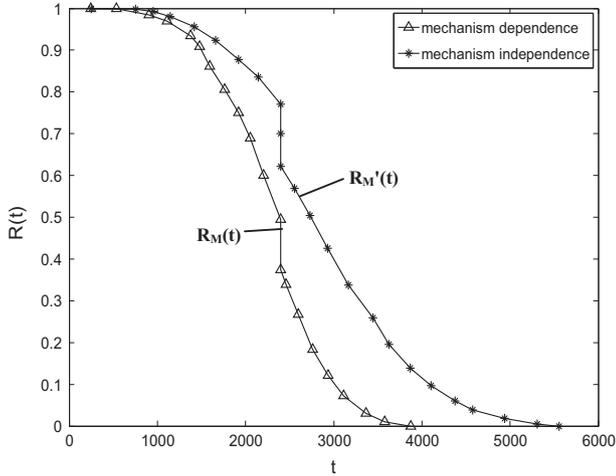


Fig. 25. Reliability function of M (system).

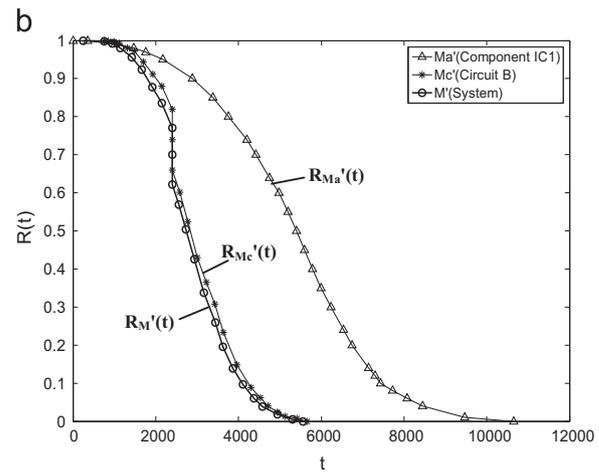
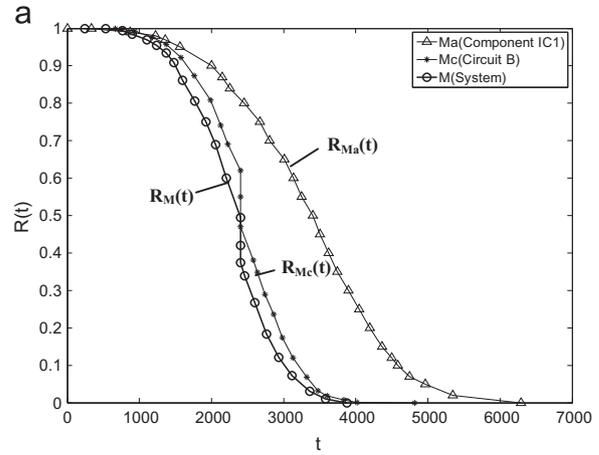


Fig. 27. Comparison of reliability function of IC<sub>1</sub>, circuit B and system. (a) Considering mechanism dependence. (b) Assuming mechanism independence.

distribution range is smaller than range with mechanism independence assumption.

5. Conclusion

This paper proposed a formulation of reliability evaluation method for non-repairable system considering failure mechanism dependence to help the decision maker and designers in designing high reliability system with minimum cost and taking into account of failure mechanism relationship.

For the non-repairable system considered in this work, correlativity between failure mechanisms includes competition, trigger, acceleration, inhibition and accumulation. Reliability of the system attribute to the subsystem or component reliability after taking into account of their relationship.

Finally, with a case, system failure probability and reliability of an electronic circuit is evaluated considering failure mechanism dependence. Results show that the reliability will be quite different compare with the system reliability under the assumption of failure independence.

This work is currently being applied in some respects including reliability prediction of electronic devices in aviation and help to perform maintenance actions for systems in aerospace area. In the future work, we will extend the PPOF method to system reliability evaluation and show how the physics-of-failure method will combine into system reliability evaluation.

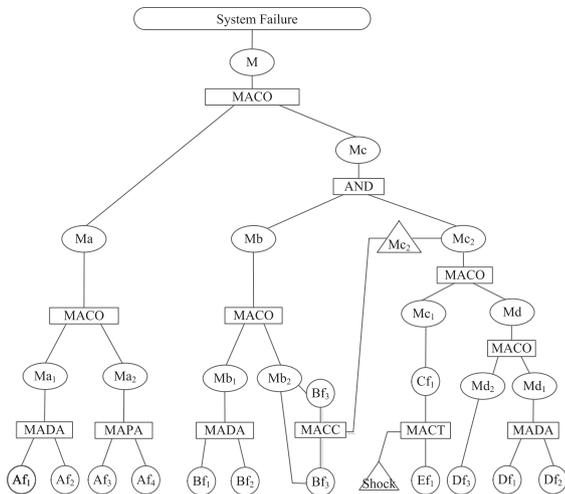
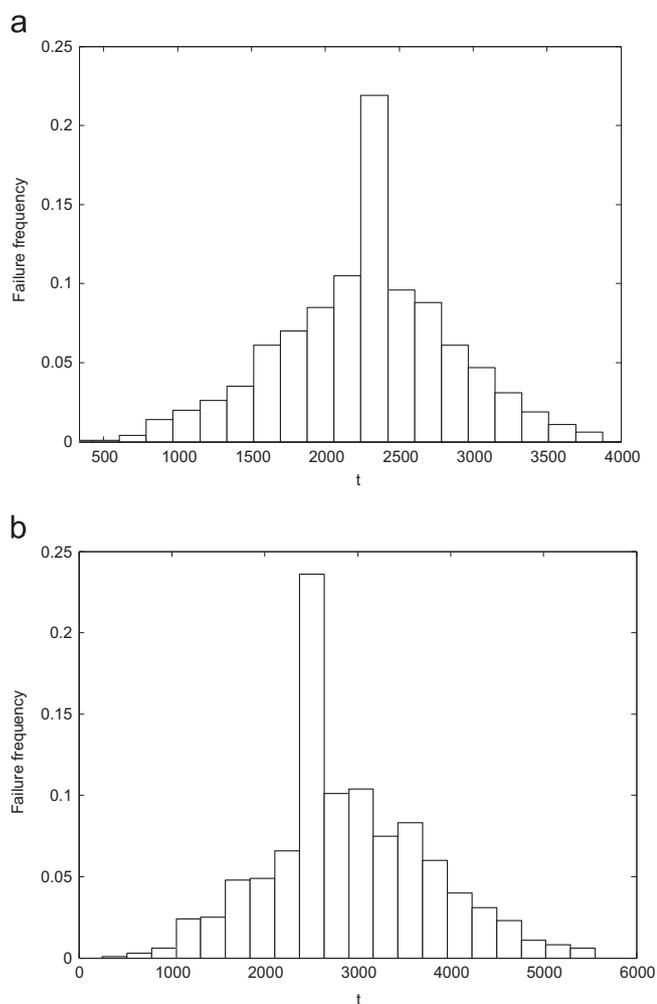


Fig. 26. System failure tree.

When considering mechanism dependence,  $R_{Ma}(t)$  decreases compared with  $R_{Ma'}(t)$ , and has more effect on system reliability  $R_M(t)$  compared to  $R_{Ma'}(t)$  affecting  $R_M(t)$ .

Based on failure time calculation results, we can draw histograms of failure frequency of system as shown in Fig. 28. Histogram can speculate the outline of failure distribution density function  $f(t)$ .

From Fig. 28, we can infer the failure distribution density function of system increases at first then decreases with time, with a spike at  $t=2400$  h. Considering mechanism dependence,



**Fig. 28.** System failure frequency in this case. (a) Failure frequency of system considering mechanism dependence. (b) Failure frequency of system with mechanism independence assumption.

## Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.ress.2015.02.002>.

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