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Research on Working Modal Identification of Electronic Equipment by Cross Spectrum

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Abstract—**Traditional technology of experimental modal analysis bases on the known input and output signals, and utilizes integrated information of both the excitation and responses to identify the modal parameter. In most of the actual conditions, only the response data is measurable while excitations are unknown. Thus modal parameter must be extracted only from responses. In allusion to this limitation, this paper puts forward to analyzing working mode of the electronic equipment by use of cross spectrum generated only by output responses.**

In order to get the response data, an integrated working modal test system has been built here. It is made up of available hardware and related software and is excited by the vibration table. This paper will use the test system and the cross-spectrum method to identify working modal parameters of a certain aviation equipment, which will then be applied to modify the finite element model (FEM). Results indicate that this test system is capable to acquire data in need, and that cross-spectrum method can identify parameters of the working mode, and also, the simulation results are in accordance with those from test. The whole system saves us much time and cost. It not only provides probability to simultaneously verify the working mode and vibration response, but also lays technical foundation for better appliance of the reliability simulation analysis method based on physics of failure (PoF) in practical projects.

I. Introduction

ACTUALLY, modal analysis is used to get the modal parameter of the structure, including natural frequency, damp ratio and mode shape. There exist two ways to conduct modal analysis. One is the analytic method which uses CAE software to solve the problem and the other is the test. In general, test results are applied for verification of the analytic method as well as modification of the finite element model.

Traditional experimental modal analysis is built on the available input and output data of the system, which utilizes integrated information of both the excitation and responses to identify modal parameters. However, it has some limitations for the difficulty to acquire complete input information for some practical projects. Accordingly, great importance has been laid on the research of working modal analysis, only based on the response data.

There are many advantages of the working modal analysis. Firstly, it merely needs the vibration response signals, which directly comes from the actual work environment; secondly, it saves cost and provides method for equipment under modal analysis with unknown input information; thirdly, modal parameters identified from the real-time response data, can be applied for online health monitoring and damage diagnosis of the device. All in all, working modal analysis makes modal analysis technology extend from static device to those in operation, thus conducting online analysis for structures unable to know the loads, and also precisely reflecting the actual dynamic characteristics.

The idea of working modal analysis has been put forward since the beginning of 1960s. Random decrement technique (RDT), first used to process response signals under ambient excitation, was also the first to be used by Cole Jr. to identify vibration parameters of the space shuttle and vehicle [1]. In 1965, Clarkson and Mercer proposed the cross-correlation function to estimate frequency response characteristics of the structure under white noise excitation, which built a framework to replace impulse response function by cross-correlation function when the excitation was not known. In 1969, time series ARMA modeling was first adopted by Akaile to process orderly random sampling data and conduct modal parameter identification under white noise excitation [2]. In 1982, HavardVold come up with PRCE method, based on the single reference complex index method [3]. In 1984, Juang first applied Eigensystem Realization Algorithm (ERA), a MIMO time domain method, into structural dynamic field [4]. In 1995, James and Came, working in SADIA national laboratory in the US, proposed NExT method, which was then used for the determination of natural frequency and damp ratio of the high-speed turbine blade [5]. Time-frequency analysis method, overcoming the shortcomings of time domain analysis and frequency domain analysis, provides a new way to identify working modal parameters and applies to both the stationary and the non-stationary excitation signals.

In domestic, vibration engineering institute of Nanjing University of Aeronautics and Astronautics (NUAA) has been working on the research of modal analysis. In addition, Chinese vibration association, noise impact vibration state key laboratory of Shanghai Jiao Tong University (SJTU) and Harbin Institute of Technology are all devoted to study the identification method of working modal parameters.

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This paper centers on cross-power spectrum based working modal analysis theory and builds a relevant test system excited by the vibration table. It also gives aviation parameter processing equipment as an example and put the theory into practice by using cross-power spectrum to identify the working modal parameters and applying the results to modify the FEM.

II. Cross-power Spectrum Based Working Mode Analysis Theory

A. System Basic Assumption

Unless stated, the system to be analyzed is linear time invariant system. Specific assumptions are as follows [6][7]:

1) Linear hypothesis. The target system is linear, which means the response signal meets linear superposition principle.
2) Time invariant. The target system is steady and stable, which means that the dynamic characteristics of the vibration system have frequency stability, and that finite excitation produces finite response. It meets the condition of Laplace and Fourier transforms.
3) Observability. The input and output results of target system have enough information to describe its characteristic model.

With above hypotheses, the linear oscillatory differential equation of multi-degree-of-freedom system in modal analysis can be described as:

$$M\ddot{x}+C\dot{x}+Kx=f(t) \tag{1}$$

B. Modal Expansion of the Transfer Function or the Frequency Response Function (FRF)

After defining symbol expression, reliability modeling can be conducted based on structure and function of system. Process of reliability modeling could be approximately divided into three steps:

Transform (1) by Laplace's theorem with the initial value 0:

$$\left(s^2M+sC+K\right)X(s)=F(s) \tag{2}$$

Transfer function matrix:

$$[\mathrm{H(S)}]=\frac{\{X(s)\}}{\{F(s)\}}=\frac{1}{[M]s^2+[C]s+[K]} \tag{3}$$

Express the matrix with residue by expanding it as the pole:

$$[\mathrm{H(S)}]=\sum_{\mathrm{r}=1}^{N}\left\{\frac{[A_r]}{s-p_r}+\frac{[A_r^*]}{s-p_r^*}\right\} \tag{4}$$

$[A_r]$ and $[A_r^*]$ are the residue matrixes of the pole p_r and p_r^*, respectively. Taking $s=j\omega$, the transfer function will be transformed into FRF.

The residue matrix $[A_r]$ can be expressed with the modal parameter:

$$[A_r]=\frac{\psi_{\mathrm{r}}\psi_{\mathrm{r}}^T}{a_r}=\frac{1}{a_r}\begin{bmatrix}\psi_{1\mathrm{r}}\psi_{1\mathrm{r}} & \psi_{1\mathrm{r}}\psi_{2\mathrm{r}} & \cdots & \psi_{1\mathrm{r}}\psi_{\mathrm{Nr}}\\ \psi_{2\mathrm{r}}\psi_{1\mathrm{r}} & \psi_{2\mathrm{r}}\psi_{2\mathrm{r}} & \cdots & \psi_{2\mathrm{r}}\psi_{\mathrm{Nr}}\\ \vdots & \vdots & \ddots & \vdots\\ \psi_{\mathrm{Nr}}\psi_{1\mathrm{r}} & \psi_{\mathrm{Nr}}\psi_{2\mathrm{r}} & \cdots & \psi_{\mathrm{Nr}}\psi_{\mathrm{Nr}}\end{bmatrix} \tag{5}$$

As can be seen, each row or column of the residue matrix contains the r$^{\mathrm{th}}$ modal vector. Thus modal parameters can be extracted by solving any row or column of the FRF matrix [8].

C. Relations between FRF and Cross-power Spectrum

Cross-power spectrum is the product of Fourier transforms of test response signal and the reference response signal. Its amplitude is the product of the amplitude of test response signal and the reference. Its phase is the difference of the phase of test response signal and the reference [9]. This paper will state the relations between FRF and cross-power spectrum, and demonstrate the feasibility of working modal parameter identification based on cross-power spectrum.

In order to describe response characteristics of the system, it defines impulse response function $h(t)$ in the time domain and FRF $H(\omega)$ in the frequency domain, which are Fourier pair. Then,

$$H(\omega)=\int_{-\infty}^{\infty}h(t)e^{-i\omega t}dt \tag{6}$$

Cross-correlation function of the excitation and response is:

$$R_{fx}(\omega)=E[f(t)x(t+\tau)]=\int_{-\infty}^{\infty}h(\theta)R_f(\tau-t)d\theta \tag{7}$$

The cross-power spectrum can be extracted by transforming the cross-correlation function via Fourier transform:

$$S_{fx}(\omega)=\int_{-\infty}^{\infty}R_{fx}(\tau)e^{-i\omega\tau}d\tau=\int_{-\infty}^{\infty}h(\theta)e^{-i\omega\tau}S_f(\omega)d\theta=H(\omega)S_f(\omega) \tag{8}$$

It indicates that the cross-power spectrum is the product of excitation spectrum and the FRF of the system. If the cross-power spectrum and excitation spectrum are already known, the FRF can be obtained. When the excitation is white noise, its spectrum $S_f(\omega)$ is a constant. Thus the difference between cross-power spectrum and the FRF merely lies in a constant. Both their poles contain the information of frequency and damp, while the residue is in proportion to the modal model. Therefore, working modal parameters based only on the response data are able to be calculated by using cross-power spectrum of the test response signal and the reference [10].

III. A VIBROSTAND-DRIVEN WORKING MODAL TEST SYSTEM

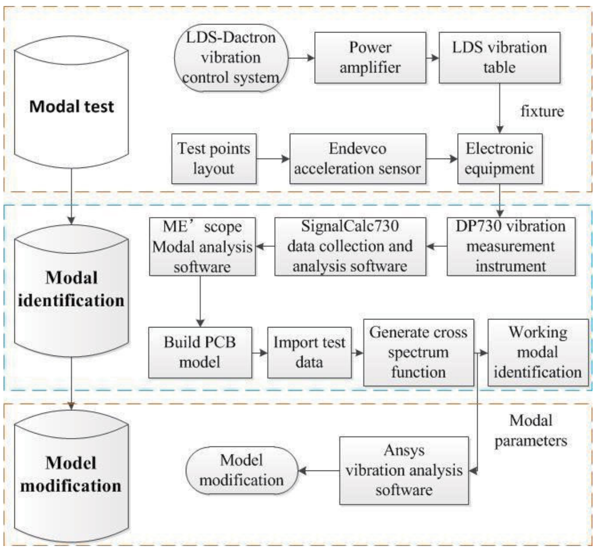


Fig. 1. The vibrostand-driven working modal test system.

Fig. 1 shows an integrated working modal test system, which is made up of available test equipment and related software. The principle can be described as follows: firstly, set the vibration spectrum in the vibration control system and export the signal into a power amplifier to drive the vibration table; secondly, stick piezoelectric acceleration sensors to the PCB, fix the electronic equipment to vibration table by the fixture, and feed back the acceleration signal via a vibration measurement instrument to the data collection and analysis software; sample and record real-time data and export it to modal analysis software to identify working modal parameters, including frequency, damp, residue, mode shape; finally, modify the FEM to be more actual and credible, according to these results, by correcting material parameters or adjusting grid number and quality.

IV. WORKING MODE ANALYSIS OF THE AVIATION PARAMETER PROCESSING EQUIPMENT

Taking a certain aviation parameter processing equipment for an example, this chapter makes use of the system built above to conduct modal test, modal analysis as well as model modification, and introduce each step in detail.

A. Brief Introduction of the Aviation Parameter Processing Equipment

The aviation parameter processing equipment is mainly responsible for collecting data and processing signals. It has 4 input bus, 6 output bus, and is made up of the equipment box, relay box, 12 PCBs, 1 motherboard and the aerial plug. The five main types of modules are power module, digital signal module, analog signal module, CPU module as well as power conversion module. Now we will utilize the working modal analysis system to test the acceleration response signals of each module, and extract working modal parameters according to the response data, which will then be used to modify the FEM.

B. Working Mode Test

Process of working mode test could be approximately divided into four steps:

1) Layout of the test points: As the target of this test is PCB with simple structure, we adopt equal distribution method to set the sensors;
2) Installation of the sensors: There are mainly four ways to connect sensors and the test object: by bolt, adhesives, wax or permanent magnet. The Endevco7703A-50 acceleration sensor in this test is with no magnetic base and requires high upper limited frequency. So we fix them via hard adhesives;
3) Operation and data collection: Set broadband random vibration spectrum which is very close to the white noise, and make sure it is loaded on the vibration table and that it is along the normal direction of the PCB. Via the fixture, excitation signal will be passed on from the vibration table to the electronic equipment (see Fig. 2). Sample and record real-time data in the data collection software, as shown in Fig. 3;
4) Data processing: Save the data in a format that can be recognized by modal analysis software and make preparation for modal parameter identification.

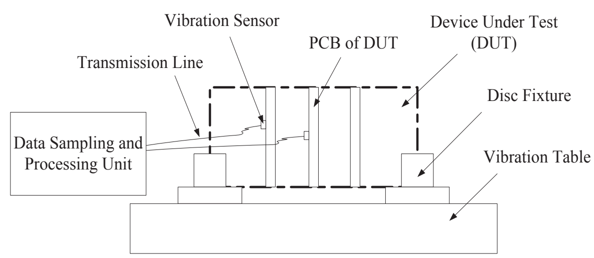


Fig. 2. Equipment installation schematic diagram.

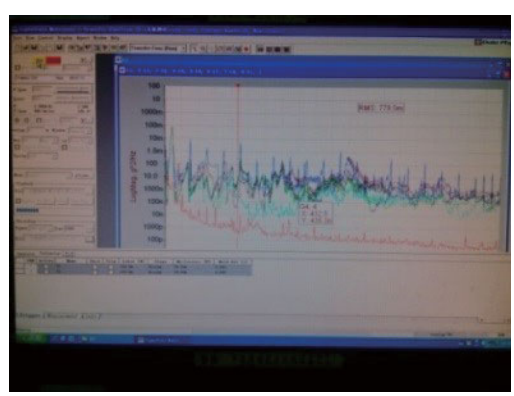


Fig. 3. Data collection interface.

C. Identification of the Working Modal Parameter

Here we use ME'scope as the working modal analysis software. The detailed parameter identification process is as follows:

1) Create a PCB model, as shown in Fig. 4;
2) Import the test data, among which the time-domain wave curves of some channel can be seen in Fig. 5;
3) Generate cross-power spectrum function. Fig. 6 shows the cross-power spectrum curves of two test points;
4) Identify working modal parameters in accordance with the cross-power spectrum. Determine the modal order by amplitude estimation or imaginary part estimation of the Modal Peaks Function estimation method; select the Peaks method to fit frequency and damp. Attention here, we may fit the parameters in a wide frequency range if the orders are dispersedly distributed, while it is supposed to divide the frequency band into several when the influence of modal coupling or interference cannot be neglected; choose the Peaks method to extract residue and mode shape. When needed, it is able to animate the working mode shape after matching the freedom of cross-power spectrum and test points of the PCB model.

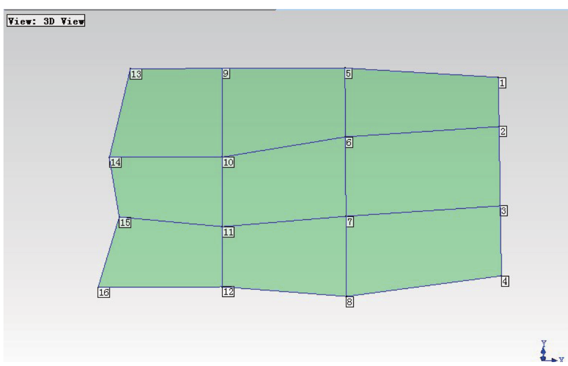


Fig. 4. PCB model.

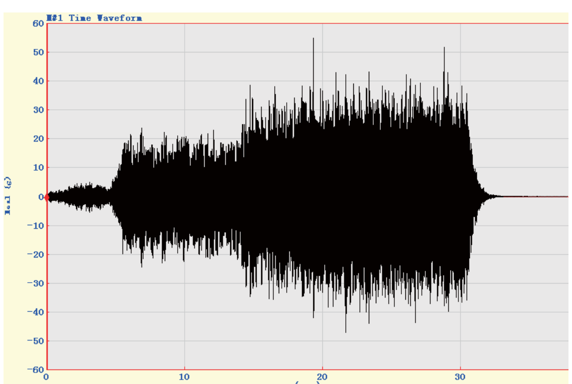


Fig. 5. Time-domain wavecurves of some channel.

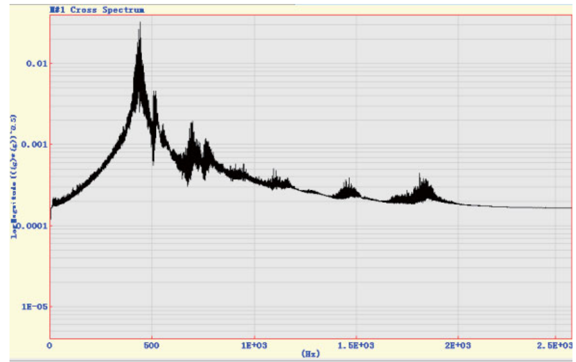


Fig. 6. Cross-power spectrum curves.

D. Modification of the Finite Element Model

Modify the FEM in accordance with the modal analysis results by the following four ways:

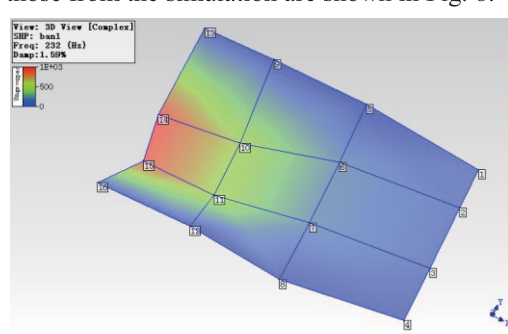
1) Correct material parameters of the modules, such as elasticity modulus, mass;
2) Correct material parameters of the components by observation and measurement;
3) Adjust local components and complement some others that are initially neglected;
4) Control and adjust the number and quality of the grid.

TABLE I shows test and simulation results of the first three modal frequency and mode shape of the power module. It can be seen that the calculated results are in good agreement with the measured ones in the experiments.

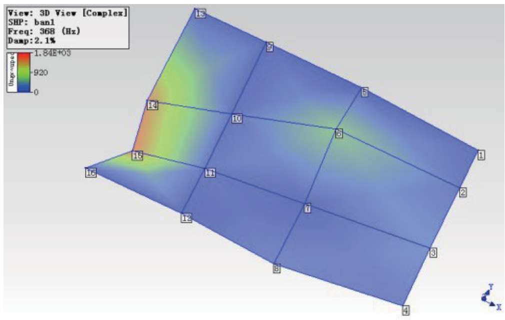
TABLE I
FIRST THREE MODAL FREQUENCY AND MODE SHAPES OF THE POWER MODULE

Object	Order	Frequency of test	Frequency of simulation	Error(%)
Power module	First	232	241	3.9
	Second	368	349	5.2
	Third	518	477	7.9

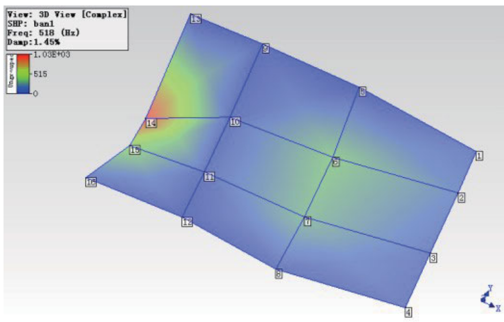
Fig. 7 displays the first three mode shapes from the test, while those from the simulation are shown in Fig. 8.



a） The first mode shape.

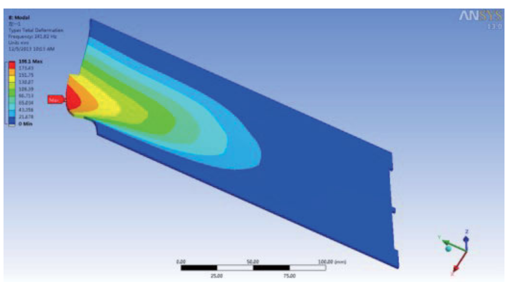


b)　The second mode shape.

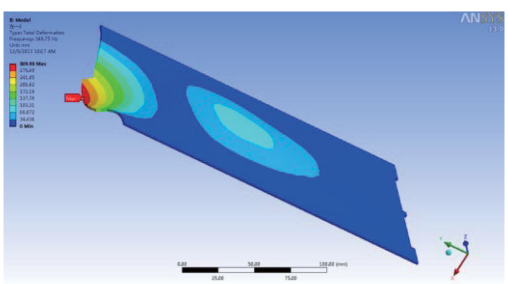


c)　The third mode shape.

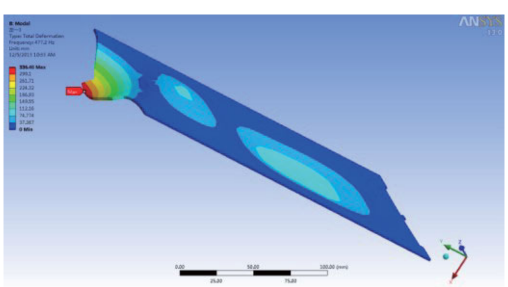
Fig. 7. The first three mode shapes from the test



a)　The first mode shape.



b)　The second mode shape.



c)　The third mode shape.

Fig. 8. The first three mode shapes from the simulation

V. Conclusion

Through field test, modal analysis as well as FEM modification of the avionic device, this paper gives detailed theoretical demonstration and empirical verification of the following conclusions:

1) With only the response data, cross-power spectrum based working modal analysis method is able to identify the modal parameter of electronic equipment;
2) The vibrostand-driven working modal analysis system can complete collection and processing of the relevant data;
3) There is good consistence between test and simulation after modifying FEM by the modal results.

By using of this method, it no longer needs to test the input excitation, saving test cost and time, and it enables simultaneous verification of the working mode and vibration response, and also provides technical foundation for the better application of the reliability simulation analysis method based on PoF in practical projects.

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